An Abstract CNF-to-d-DNNF Compiler Based on Chronological CDCL

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The Task: Knowledge Compilation

$$F \longrightarrow \mathrm{KC}(F) \equiv F$$

The Task: CNF-to-d-DNNF Compilation

F → $\mathsf{KC}(F) \equiv F$

this work

conjunctive normal form (CNF)

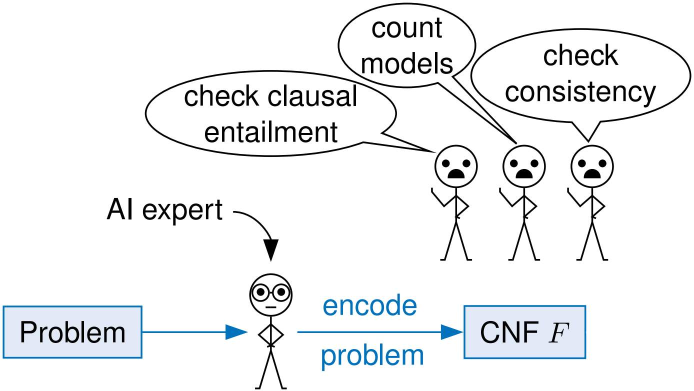
deterministic decomposable negation normal form (d-DNNF)

Deterministic Decomposable Negation Normal Form

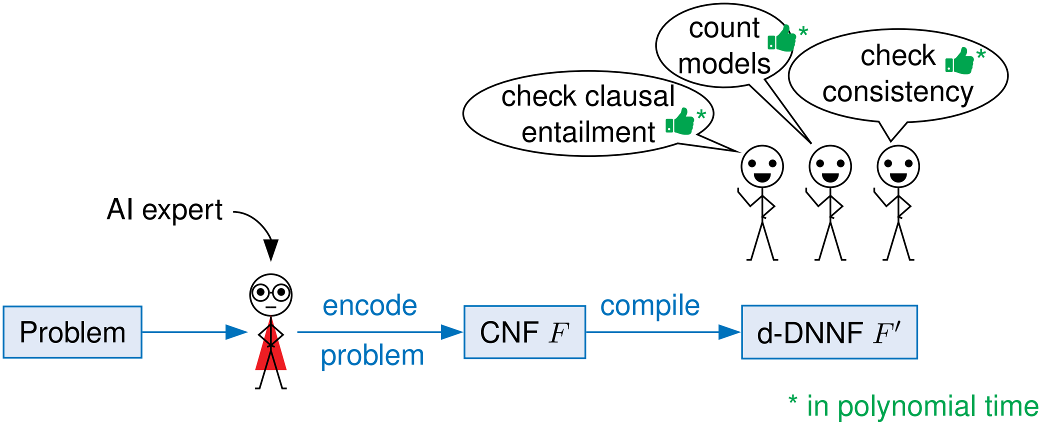
d-DNNF: $F = (a \wedge (b \vee (\neg b \wedge c)) \wedge d) \vee (\neg a \wedge b)$

- negations in front of variables
- for all conjunctions: conjuncts do not share variables
- for all disjunctions: disjuncts are pairwise contradicting

CNF or d-DNNF — Why Care?



CNF or d-DNNF — Why Care?



CNF vs. d-DNNF — the Model Counting Case

CNF: $F = (\neg a \vee b \vee c) \wedge (\neg a \vee d) \wedge (a \vee b)$

d-DNNF: $F' = (a \wedge (b \vee (\neg b \wedge c)) \wedge d) \vee (\neg a \wedge b)$

CNF vs. d-DNNF — the Model Counting Case

CNF: $F = (\neg a \lor b \lor c) \land (\neg a \lor d) \land (a \lor b)$

$\#F = ?$ not that easy

d-DNNF: $F' = (a \land (b \lor (\neg b \land c)) \land d) \lor (\neg a \land b)$

$\#F' = (1 \cdot (1 \cdot 2^1 + 1 \cdot 1 \cdot 2^0) \cdot 1) \cdot 2^0 + (1 \cdot 1) \cdot 2^2 = 7$

What About State-of-the-Art Tools?

	CDCL	Backtracking	Formula construction	Decomposition
c2d[1]	✔	non-chronological	record trace	static
dSharp[2]	✔	non-chronological	record trace	dynamic
D4[3]	✔	non-chronological	record trace	dynamic
ACD[4]	✔	chronological	conjoin DSOPs	dynamic

[1] Darwiche, ECAI'04
[2] Muise et al., CANAI'12
[3] Lagniez & Marquis, IJCAI'18
[4] Abstract CNF2dDNNF — this work

Contributions

- ABSTRACT CNF2DDNNF — a calculus for CNF-to-d-DNNF compilation
- Combination of model enumeration with chronological CDCL with dynamic component analysis
- Formal correctness proof
- First CNF-to-d-DNNF compiler based on chronological CDCL

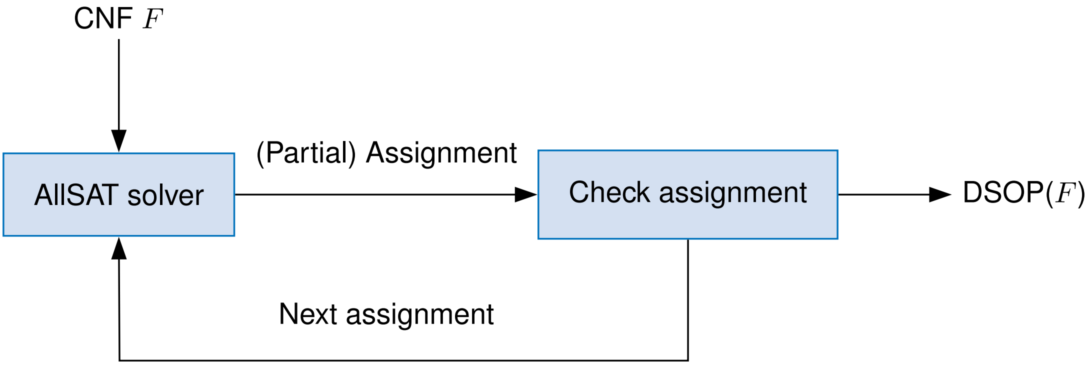
Outline of the Rest of the Talk

The working of ABSTRACT CNF2dDNNF

ABSTRACT CNF2dDNNF by an example

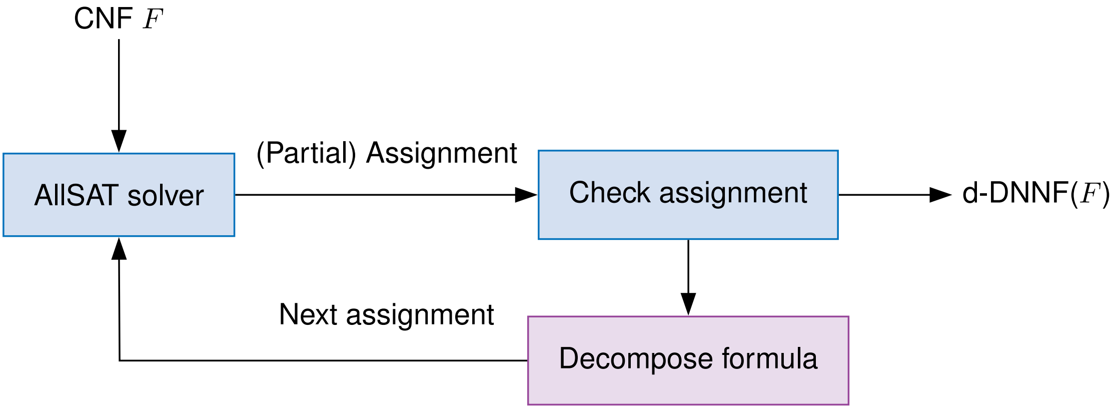
Recap and ideas for further work

The Main Idea



DSOP = disjoint sum-of-products

The Main Idea



From DSOP to d-DNNF

$$F = (\neg a \lor b \lor c) \land (\neg a \lor d) \land (a \lor b)$$

$$\text{DSOP}(F) = (\underline{a} \land b \land \underline{d}) \lor (\underline{a} \land \neg b \land c \land \underline{d}) \lor (\neg a \land b)$$

From DSOP to d-DNNF

$$F = (\neg a \vee b \vee c) \wedge (\neg a \vee d) \wedge (a \vee b)$$

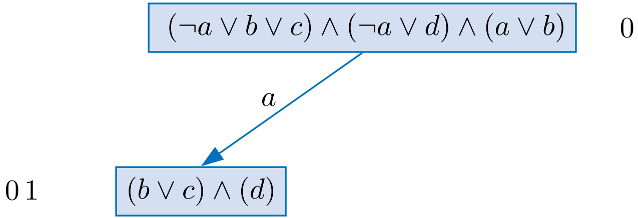
$$\text{DSOP}(F) = (\underline{a} \wedge b \wedge \underline{d}) \vee (\underline{a} \wedge \neg b \wedge c \wedge \underline{d}) \vee (\neg a \wedge b)$$

$$\text{d-DNNF}(F) = (\underline{a} \wedge (b \vee (\neg b \wedge c)) \wedge \underline{d}) \vee (\neg a \wedge b)$$

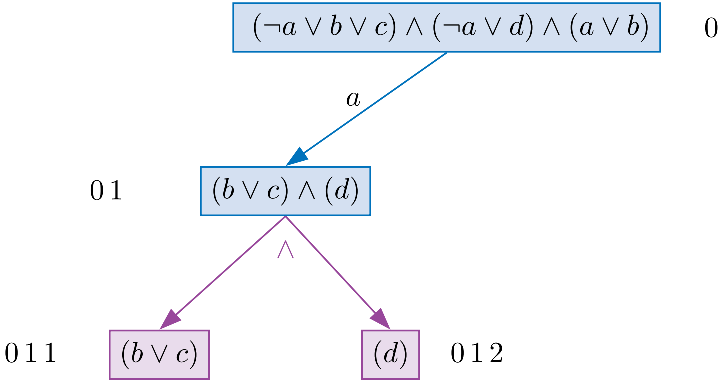
CNF-to-d-DNNF Compilation

$$(\neg a \lor b \lor c) \land (\neg a \lor d) \land (a \lor b) \qquad 0$$

CNF-to-d-DNNF Compilation



CNF-to-d-DNNF Compilation



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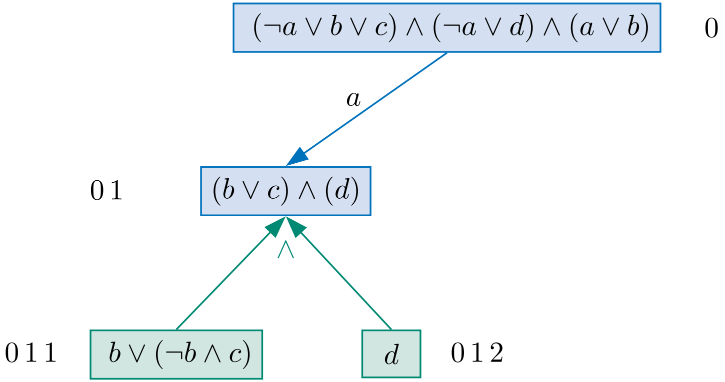

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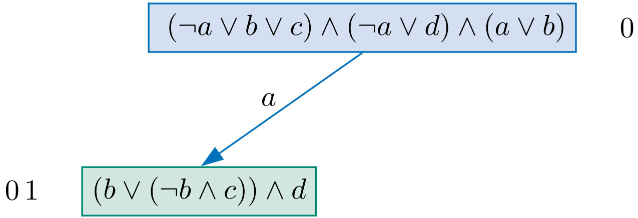

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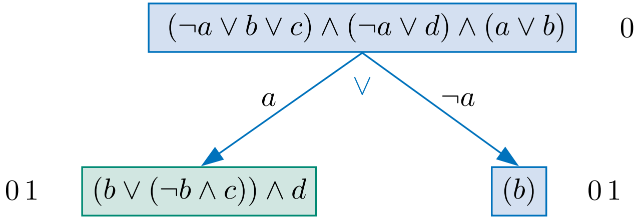

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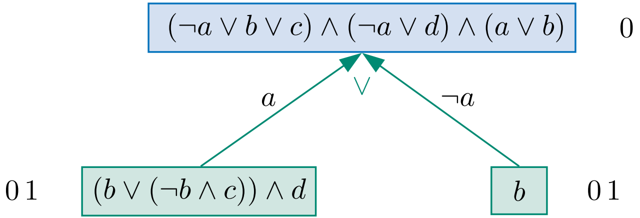

CNF-to-d-DNNF Compilation

$$(a \land (b \lor (\neg b \land c)) \land d) \lor (\neg a \land b) \qquad 0$$

Model Counting in d-DNNF

(1) $F = G \wedge H \implies \#F = \#G \cdot \#H$
provided $\mathsf{var}(G) \cup \mathsf{var}(H) = \mathsf{var}(F)$ and $\mathsf{var}(G) \cap \mathsf{var}(H) = \emptyset$

(2) $F = C \vee D \implies \#F = 2^{|\mathsf{var}(F)|-|\mathsf{var}(C)|} + 2^{|\mathsf{var}(F)|-|\mathsf{var}(D)|}$
provided $C \wedge D \equiv \bot$

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(1) $F = (a \vee b) \wedge (c \vee d) \implies \#F = 3 \cdot 3 = 9$

$\mathcal{M}(F) = \{abcd, abc\neg d, ab\neg cd, a\neg bcd, a\neg bc\neg d, a\neg b\neg cd, \neg abcd, \neg abc\neg d, \neg ab\neg cd\}$

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$$\mathcal{M}(F) = \{abcd, abc\neg d, ab\neg cd, a\neg bcd, a\neg bc\neg d, a\neg b\neg cd, \neg abcd, \neg abc\neg d, \neg ab\neg cd\}$$

(2)
$$F = a \vee (\neg a \wedge b \wedge c) \implies \#F = 2^2 + 2^0 = 5$$
$$\mathcal{M}(F) = \{abc, ab\neg c, a\neg bc, a\neg b\neg c, \neg abc\}$$

Model Counting in d-DNNF

$$F = \underline{(a \land (b \lor (\neg b \land c)) \land d)} \lor \underline{(\neg a \land b)} \quad (2)$$

$$\#F = \#(\underline{a} \land \underline{(b \lor (\neg b \land c))} \land \underline{d}) \cdot 2^0 + \#(\underline{\neg a} \land \underline{b}) \cdot 2^2 \quad (1)$$

$$= [\#(a) \cdot \#(\underline{b} \lor \underline{(\neg b \land c)}) \cdot \#(d)] \cdot 2^0 + [\#(\neg a) \cdot \#(b)] \cdot 2^2 \quad (2)$$

$$= [1 \cdot [\#(b) \cdot 2^1 + \#(\underline{\neg b} \land \underline{c})] \cdot 2^0] + [1 \cdot 1] \cdot 2^2 \quad (1)$$

$$= [1 \cdot [1 \cdot 2^1 + [\#(\neg b) \cdot \#(c)] \cdot 2^0] + 2^2$$

$$= [1 \cdot [1 \cdot 2^1 + [1 \cdot 1] \cdot 2^0] + 2^2 = 7$$

This work

- CNF-to-d-DNNF compilation calculus
- Enumerative approach
- Based on chronological CDCL
- Formal proof of correctness

Ideas for future work

- Implementation (of proof of concept)
- Target compact formula representation
- Effect of decomposition on dual reasoning
- Investigate decomposability of formulae

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Thank you for your attention ☺